the proof of the existence or non-existence of simple groups of an odd order or of order $p^{\alpha} q^{\beta}$, $p$ and $q$ being prime numbers; the superior limit of transitivity of primitive groups that do not contain the alternating group; the simplification of the methods of proving the solvability or the insolvability of a group, etc.

---

# REPLY TO PROFESSOR FISK'S CRITICISM OF A CERTAIN FEATURE OF NICHOLSON'S CALCULUS.

By J. W. NICHOLSON, A. M., LL. D., Professor of Mathematics, Louisiana State University, Baton Rouge, La.

In the March number of the *Bulletin* is a brief review of my Calculus, by Professor Fiske, of which the following is an extract:

"In another note ($A_3$) at the end of the work the author critizes the grounds assigned by Byerly and by Rice and Johnston for making $d(dx)=0$. He contends that the differential of $dx$ is zero, because $dx$ as a variable is independent of $x$. This, of course, is not sound. If a variable $y$ is independent of another variable $x$, it is true that we may still write

$$dy=\frac{dy}{dx}dx;$$

but the coefficient of $dx$ is not a partial derivative, and $dy$, therefore, instead of being zero is indeterminate. In order that $d(dx)$ may be zero, we must assume that $dx$ takes the same value for all values of $x$. This assumption, however, does not prevent our varying $dx$ from one instant to another in a perfectly arbitrary manner."

As the question involved is an interesting and important one, and believing that Professor Fiske had not fairly presented my discussion of the point at issue, I wrote a brief reply to the above criticism, and sent it to the *Bulletin* for publication. Several weeks thereafter my reply was returned to me without publication and with the following additional stricture:

"The author fails to realize that if $dy=0$ when $x$ goes from $x$ to $x+dx$ then $y$ is not completely independent of $x$, but has such a dependence that it does not alter when $x$ alters."

The question involved is not whether $dx$ is a quantity whose total differential is 0, but whether it is a quantity whose differential *with respect to x* is 0.

Of course if $x$ and $y$ are two variables which are independent of each other, and $dx$ be the total differential of the one and $dy$ that of the other, and $\frac{dy}{dx}$ is understood to mean the ratio of these differentials, $\frac{dy}{dx}$ is not 0 but indeterminate, as Professor Fiske says. Or again, under the same hypothesis, "if $dy=0$

when $x$ goes from $x$ to $x+dx$'' $y$ would depend on $x$ in the manner indicated by the last criticism.

But the point in question comes up in proving that $\frac{d}{dx}\left(\frac{dy}{dx}\right)$, *i. e.* the differential coefficient of $\frac{dy}{dx}$ with respect to $x$, $=\frac{d^2y}{dx^2}$; and in this demonstration we have no occasion to consider whether $dx$ is a quantity whose total differential is 0 or not. It is a differentiation with respect to $x$ which is indicated by $\frac{d}{dx}$; and it is to be demonstrated that if this operation be performed on $\frac{dy}{dx}$, on the hypothesis that this symbol may be treated as a fraction whose terms are $dy$ and $dx$, and the understanding that *$d^2y$ represents the differential of $dy$ with respect to $x$*, the result is $\frac{d^2y}{dx^2}$. Thus:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{dx\frac{d}{dx}(dy)-dy\frac{d}{dx}(dx)}{dx^2}=\frac{d^2y}{dx^2},$$

because $d^2y=\frac{d}{dx}(dy).dx$, by definition, and $\frac{d}{dx}(dx)=0$, *for the same reason that the differential coefficient with respect to $x$ of any variable which is independent of $x$ is 0.* $\frac{d}{dx}(dx)$ does not mean the ratio of any variation that $dx$ may be supposed to undergo, to $dx$, but the ratio of the variation which $dx$ undergoes *in consequence of a variation in $x$*, to $dx$. To say that $\frac{d}{dx}(dx)=0$ is not therefore to say that $dx$ is a constant, but merely that it undergoes no variation in consequence of a variation in $x$. Indeed, $dx$ may have any value of $x$, and is therefore a variable independent of $x$, and being *independent*, it may be regarded and treated as an *absolute constant* except in cases where the independence of $x$ would thereby be destroyed, as shown in my Calculus.

Precisely the same considerations are involved in the derivation of the equation $d^2y=f''(x)dx^2$ from $dy=f'(x)dx$.

The reply to Professor Fiske is therefore that in his equation, $dy=\frac{dy}{dx}dx$, for the case before us, viz: $d^2x=\frac{d}{dx}(dx).dx$, the coefficient of $dx$ *is a partial derivative.*

The reply to the last criticism of the *Bulletin* is that in the case before us the $dy$ is not any variation that $y$ may be supposed to undergo while $x$ varies from $x$ to $x+dx$, but the variation which $y$ undergoes in consequence of this variation in $x$, and this of course must be 0 if $y$ is independent of $x$, whatever may be the value of $y$ as $x$ goes from $x$ to $x+dx$.

The criticism in the *Bulletin* is therefore based upon a misconception of

the author's meaning, and is due to an apparent failure on the part of Professor Fiske to realize that the question is not what must be in order that $d(dx)$ may be 0, but what is in order that $\frac{d}{dx}(dx)$ may be 0.

# DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

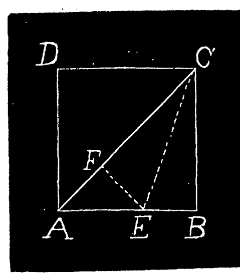
### ARITHMETIC.

96. Proposed by RAYMOND SMITH, Tiffin, Ohio.

How many acres in a square field whose diagonal is 10 rods longer than the side ?

I. Solution by J. F. TRAVIS. Student at Ohio State University, Columbus, O.; EDWARD R. ROBBINS, Master of Mathematics, Lawrenceville School, Lawrenceville. N. J.; J. SCHEFFER. A. M., Hagerstown, Md.; F. R. HONEY. Ph. B., New Haven. Conn.; M. E. GRABER, Mt. Eaton; O.; WALTER HUGH DRANE. Professor of Mathematics, Jefferson College, Washington, Miss.; and JOSIAH H. DRUMMOND, LL. D., Portland, Me.

Let $ABCD$ be the square field, and $AC$ its diagonal. On $AC$ lay off $CF$ equal to $BC$. At $F$ erect $FE$ perpendicular to $AC$ and intersecting $AB$ in $E$. Draw $EC$. Then in the right triangles $CFE$ and $CBE$, $CB$ equals $CF$, by construction and $CE$ is common. Hence, $FE$ equals $EB$. In the right triangle $AFE$, the angle $FAE$ is equal to 45°. Hence, the angle $FEA$ equals 45°. Hence the side $AF$ equals the side $FE$. Then




$$AB=(AE+EB)=[\sqrt{(AF^2+FE^2)}+EB]$$
$$=[\sqrt{(2EF^2)}+EB]=(EF\sqrt{2}+EB)=(\sqrt{2}+1)EB.$$

But $EB=AF=10$ chains. $\therefore$ $AB=10(\sqrt{2}+1)$, and area of the field$=AB^2=100(\sqrt{2}+1)^2=100(3+2\sqrt{2})=582.8427$ square rods, or 3.642 acres.

II. Solution by M. A. GRUBER, A. M., War Department. Washington, D. C.; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics, Chester High School, Chester, Pa.; and P. S. BERG, Superintendent of Schools, Larimore, N. D.

We will solve generally by making $a$=the excess of the diagonal over the side.

Let $x$=side of square field. Then $2x^2=(x+a)^2$.

Solving this equation for $x$, gives, $x=a(1\pm\sqrt{2})$.

$\therefore$ area$=x^2=a^2(3\pm2\sqrt{2})$.

Now substituting 10 for $a$, and we obtain

Area$=100(3\pm2\sqrt{2})=582.8427+$ square rods, or $17.157+$ square rods,
$=3.6429+$ acres, or $.10723+$ acres.